Industry Structure and Wage Inequality during Pandemic

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Abstract

The COVID-19 pandemic has accelerated the switch to E-commerce and widened wage inequality between the online and offline sectors. Motivated by these observations and based on the workhorse SIR model, this paper develops a framework with online and offline sectors to explain the mechanism that how lockdown policy affects the consumer's purchasing behavior and wage inequality between the online and offline sector. I show that as the tightening of lockdown policy, the wage inequity also increase. The crowding-out effects is sharped in the process of COVID-19 period because of lockdown policy. This echoes what happen in the onset stage of pandemic on the consumption markets in China.

1 Introduction

The COVID-19 offers a rare opportunity to understand how the pandemic shocks may affect aggregate economic outcomes and how industrial structure evolves in the era of pandemics. One important difference between the COVID-19 and the epidemics in history is that consumers can now buy online even during lockdown. As a result, even though the COVID-19 pandemic has brought massive and unexpected life twists, it did not grind everything to a halt. Surprisingly, the market value of internet company has nevertheless increased dramatically from the onset of the pandemic. This is more pronounced for China which has implemented strict lockdown policies[1], and experienced unprecedented growth of online sales in the last decades[2]. As a result, even though the pandemic might not last long, its impacts on people's consumption behavior, labor market, and industrial structure may continue and far exceed its short-term effect.

Motivated by the above observations, this paper builds up a two-sector model and attempts to examine: how would E-commerce shape the optimal lockdown policies and what implications would lockdown policies bring on industry structure and wage inequality? In the model, the consumers choose whether to buy from the online sector or the offline sector. There are two channels through which the pandemic affects economic outcome. The first effect is the "income effect".

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[1]Recently, a spectrum of events in Shanghai has proved the effectiveness of lockdown policy wielding by the Chinese government see Chen et al. [2022]

[2]The market value of website establishments hit new historical record during 2020.

If the economy suffers from a negative shock, both the online and offline sectors are affected by the income decline. The second effect is the "substitution effect". Both the lockdown policy and the increase of the infected population raise the consumer's relative trade cost of buying offline, which causes the production structure to shift toward the online sector. This in turn increases the wage premium of the online sector as labor cannot move freely across sectors.

My research is related to two directions of research: the first line of inquiry discusses how pandemic shocks may be translated into demand shocks and the implications for macroeconomic policies, such as Guerrieri et al. [2022] , Baqaee and Farhi [2022], and Woodford [2022]. I contribute to the above literature by shedding light on the impacts of pandemic shocks on demand structure, industrial structure and wage inequality. In the present paper, wage inequality is driven by switching in the consumer's consumption behavior during the pandemics.

The second mainly concentrates on the pandemic dynamics and economy outcomes which contributes to the recent development of the SIR model proposed by Kermack et al. [1927] from the perspective of parameters estimation Atkeson et al. [2020], social network structure Berger et al. [2020] and optimal policy under the framework of SIR (e.g. Alvarez et al. [2021], Jones et al. [2021], Farboodi et al. [2021], Rowthorn and Toxvaerd [2012], Eichenbaum et al. [2021], Acemoglu et al. [2021], Barro et al. [2020], Eichenbaum et al. [2021], Hall et al. [2020], Chari et al. [2021], Baldwin and di Mauro [2020], Morton and Wickwire [1974], and Hansen and Day [2011].) I differ from the previous studies in two aspects.

First, in contrast to the above literature which model the social planner concentrates on the population dynamics and set that as the representation social welfare (Alvarez et al. [2021], Acemoglu et al. [2021]), the present paper models each agent's utility function as an expected utility over the states. My approach not only allows us to evaluate the role of consumer's risk aversion, but also delivers an explicit and more intuitive expression.

The typical approach in the epidemiology literature is to study the dynamics of the pandemic, for infected, deaths, recovered, as functions of some exogenous chosen diffusion parameters, which are in turn related to various policies, such as the partial lockdown and other measures of diffusion mitigation, and where the diffusion parameters are stratified by age and individual natures (see Acemoglu et al. [2021]). I differ from these studies in two ways: first, I place the planning problem within the general equilibrium framework where the consumers and firms chooses their optimal consumption and production behavior respectively. I construct an expected utility function that shows risk aversion in the choice between labor supply and consumption. Due to the introduction of risk aversion, my paper suggests a more conservative attitude toward lockdown policy as lock down policy reduces the agent's expected utility when the optimal target of central government integrates the personal utility and case fatality rate (CFR) simultaneously. On the firm side, I set up a monopoly firm owned by other workers of the economy which wields linear technical production function. Labor cannot enter into each department freely, so the wages are heterogeneous. This resonates the core mechanism I want to explain in this paper rather than channels shown in .

Second, in ensuing part of baseline model, I consider a model with industrial structure. In a similar spirit and market structure of Eaton and Kortum [2002], I discuss the problem of inequality i.e. the wage gap of these two sectors in the economy induced by sector heterogeneity of productivity and market share. The source of product price difference derives from technology idiosyncratic distribution and individual preference variation due to the current pandemic situation. The numerical result is indicative of my inequality enlarging mechanism: in normal times, the wage difference is only decided by technology and personal preference, nevertheless during the episode of pandemic, the inequality is enlarged by the lockdown policy, which resonates the concern of wielding lockdown with more caution.

My article is organized as follows. I set up baseline model in Section 2, and give the optimal planning problem in the ensuing part. In Section 3, I show the streamlined model with industrial structure to explain my mechanism of wage inequality and structure. I present the numerical results and parameters setting in the Section 4. Section 5 concludes.

2 Baseline Model

This section I use employs the workhorse SIR model to model the population dynamics in a framework with the expected utility for agents to solve a social planner problem.

2.1 SIR Model

As in Atkeson et al. [2020] and Alvarez et al. [2021], at any point in time t, the whole population $N(t)$ is divided into those susceptible $S(t)$, those infected $I(t)$ and those recovered $R(t)$, i.e.,

$$N(t) = S(t) + I(t) + R(t), \quad \forall t > 0.$$

The recovered category $R(t)$ here includes individuals who have been infected, survived the disease, and are assumed to be immune to COVID-19 within a certain period of time[3]. I normalize the initial population to $N(0) = 1$ where only those alive population are considered. The social planner can control a fraction $L(t)$ of the population, where $L(t) \leq 1$ allows us to meet a more actual situation to keep some significant departments and industries working on such as power plant, food supply vendors and groceries. The lockdown efficiency θ measures the proportion that population cannot contact others freely. If $\theta = 1$, the lockdown policy completely wields its effects on population. But the actual scenario I never harbor the ability to control all people to move in different cities to curb the transmission of virus; so I take parameter $\theta < 1$. In real world scenario, complete lockdown is implausible no matter from the perspective of consideration politics and some unavoidable cases like cross paths.

The law of dynamic of susceptible agents, infected agents, and total population follows the standard assumptions, also see Acemoglu et al. [2021] and Alvarez et al. [2021]

[3]This is a simplified assumption not backed up by conclusive evidence, but it won't affect the takeaway of the model.

$$\dot{S} = -\beta S(t)(1-\theta L(t))I(t)(1-\theta L(t)),$$

$$\dot{I} = \beta S(t)(1-\theta L(t))I(t)(1-\theta L(t)) - \gamma I(t),$$

$$-\dot{N} = D(t) = \phi(I(t))I(t),$$

$$\phi(I(t)) = [\varphi + \kappa I(t)]\gamma.$$

The parameter β captures the number that the susceptible agents contact the infected agents per unit time (day). I set the probability that people get infected from infected agents is 1 after contacting. For infected people, they can recover at the rate of γ. The death of disease $D(t)$ is defined as product of rate of death per unit time $\phi(I(t))$ and number of infected agents. The CFR $\phi \in (0,1)$ is defined as the rate of fatality of infected people, which is a linear form of infected population. It appears that the CFR manifest the direct proportion with $I(t)$, which reflects the jam effects in health care system, i.e. health care become scarce resources and some affected population cannot get those in time.

2.2 Agents' Utility, One Sector Production and Equilibrium

Susceptible agents have expected utility form and infected agents' are of certain form. The utility of each susceptible agent consists of two parts: maintaining the presented scenario, namely not be infected; and the possibility of being tracked into infected group. The utility formula is modeled as follows

$$E_t[u_s(c_t^s, n_t^s)] = p(L)u_s(c_t^s, n_t^s) + [1-p(L)]u_i(c_t^i, n_t^i)$$

where c_t^s and n_t^s are the consumption and labor supply of susceptible agents; c_t^i and n_t^i for infected people. The probability function $p(L)$ depends on direct proportion of lockdown policy $L(t)$ and the effectiveness of lockdown policy $\tilde{\alpha}$ with the form

$$p(L) = \tilde{\alpha} L(t). \tag{1}$$

The parameter $\tilde{\alpha}$ takes values between 0 and 1. If $\tilde{\alpha} = 1$, the policy is fully effective of lockdown policy, however, some contacts may still happen even under a full economic lockdown, in that case $\tilde{\alpha} < 1$.

Assume the consumption is inelastic during the period of pandemic, i.e. agents cannot make the optimal decision for c_t and n_t. Infected agents accept unemployment insurance b. Consumption and labor supply are linear form of wage $w(t)$ and lockdown policy $L(t)$ for susceptible people, which implying

$$c_t^s = (1-L(t))w(t), \quad n_t^s = 1 - L(t) \tag{2}$$

and

$$c_t^i = b, \quad n_t^s = 0. \tag{3}$$

One of policy in toolbox for remedy against the ongoing COVID-19 recession that is currently being debated of interest is fiscal stimulus. To consider the effects of fiscal stimulus in my model, I introduce a stylized government sector. Government debt that are used for subsidizing infected population is

$$B_t = I(t)b. \tag{4}$$

As Guerrieri et al. [2022], monopolistic firms produce the final good by employing labor, which implies the wage is same as price of product,

$$Y_t = N_t, \tag{5}$$

$$p_t = w_t. \tag{6}$$

Total demand comes from two parts: the consumption of susceptible agents $S(t)c_t^s$ and of infected agents $I(t)b$[4],

$$C_t = S(t)c_t^s + I(t)b. \tag{7}$$

Another side total supply, namely total labor supply N_t comes from only the susceptible people,

$$N_t = S(t)(1 - L(t)). \tag{8}$$

Define market equilibrium is

$$N_t = C_t. \tag{9}$$

Under this scenario, the wage of economy is

$$w_t = \frac{S(t)(1 - L(t)) - I(t)b}{S(t)}. \tag{10}$$

2.3 Planning Problem

The planning problem is modified version of Alvarez et al. [2021] and Acemoglu et al. [2021]. I employ an objective function of social planner. Assume that agents live forever, unless they die from the infection. The planner discounts all values at the rate $r > 0$ and with probability rate vaccine + cure ν per unit of time. Thus, the planning problem can be consist in maximizing the following present value

$$\max_L \int_0^\infty e^{-(r+v)t} \Big\{ \underbrace{S(t)u(c_t^s, n_t^s) + I(t)b}_{\text{Total Utility}} - \underbrace{\chi_d D(t)}_{\text{Loss Value}} \Big\} dt \tag{11}$$

[4]For simplification of analysis, recovered population doesn't involve into my analysis. In my calibration case, recovered population only account for a tiny part of total population.

The first part of the equation is the total utility of agents and the second part is disutility from loss value of life. In this case, HJB equation is

$$\begin{aligned}(r+v)V(S,I) = \min_{L\in[0,1]} \Big\{ & S(t)u(c_t^s,n_t^s) + I(t)b - \chi_d\phi(I(t))I(t) + \\ & - \partial_S V(S,I)[-\beta S(t)I(t)(1-\theta L(t))^2] \\ & + \partial_I V(S,I)[\beta S(t)I(t)(1-\theta L(t))^2 - \gamma I(t)] \Big\}\end{aligned} \tag{12}$$

The domain of $V(S,I)$ is $S+I \leq 1$. Following Alvarez et al. [2021], I will use the value function iteration method to solve this problem. Note the boundary of value function has an analytical formula. $V(S,0) = \frac{(1+\tilde{\alpha}bS)^2}{4\tilde{\alpha}(r+v)}$ and $V(0,I) = \frac{a^2b^2}{8\tilde{\alpha}\gamma+4\tilde{\alpha}(r+v)}I^2 + \frac{4\tilde{\alpha}\chi_d\phi+2ab}{4\tilde{\alpha}\gamma+4\tilde{\alpha}(r+v)}I + \frac{1}{4\tilde{\alpha}(r+v)}$.

3 Modified Model for Industrial Structure

This part will explain the fact that why the wage of online sector i.e. establishments based on electronic business are affected less during the onset period of pandemic.

3.1 Technology, Price and Equilibrium

Product is indexed as $j \in J$ from sector $i \in [Online,\ Offline]$. I assume the price of products of online sector is proportional to wage w_{online} and inversely for technology[5]

$$P_{online} = \left(\frac{w_{online}}{z_{online}(j)}\right) \tag{13}$$

$z_{online}(j)$ is the technology to produce j. Efficiency is a random variable generated by following distribution:

$$F_i(z) = e^{-T_i z^{-\vartheta}} \tag{14}$$

This is a workhorse distribution that is always used in literature about technology and innovation. It is related to idiosyncratic technology of online sector $z_{online}(j)$ and industry-specified parameter T_i. I assume that industry efficiency distribution is Fréchet (also called the Type II extreme value where $T_i > 0$ and $\vartheta > 1$). I treat the distributions as independent across industries. The (industry-specific) parameter T_i governs the location of the distribution. A bigger T_i implies that a high efficiency draw for any good j is more likely. The parameter ϑ (which I treat as common to all industries) reflects the amount of variation within the distribution. A bigger ϑ implies less variability.

And I normalize the offline wage $w_{offline}$ to 1, which won't affect the conclusion and is convenient to focus on the wage of online sector. In order to embody the effect of epidemic I think

[5] The microfoundation of price can be found in Eaton and Kortum [2002].

the offline product price is also affected by potential price factor $d_{offline}(I, L)$ with exponential function format. Specifically, the price of products of offline sector is

$$P_{offline} = \left(\frac{1}{z_{offline}(j)}\right) d_{offline}(I, L) = \left(\frac{1}{z_{offline}(j)}\right) e^{\alpha I + \mu L} \tag{15}$$

From this specification, I can see the price of product of offline sector increases with infected population $I(t)$ and lockdown $L(t)$ and α and μ govern the gradient that prices increases with these two variables.

I assume perfect competition, which means all the products in certain sector are set to the same price. They shopping around two sectors for the best deal, namely consumer will be free to choose the cheapest product and then I have

$$P^* = \min \left\{P_{online}, P_{offline}\right\} \tag{16}$$

The lowest price will be less than p unless all source's price is greater than that. These assumptions imply I can get following price distribution of online and offline sectors:

$$G_{online}(p) = \Pr\left[P_{online} \leq p\right] = \left(\frac{w_{online}}{z_{online}(j)}\right) \leq p \tag{17}$$

$$G_{offline}(p) = \Pr\left[P_{offline} \leq p\right] = \left(\frac{1}{z_{offline}(j)}\right) e^{\alpha I + \mu L} \leq p \tag{18}$$

So the technology distribution of these two sectors when the market is clear should be

$$G_{online} = 1 - \exp\left\{-p^{\vartheta}\left[T_{online}\left(w\right)^{-\vartheta}\right]\right\} \tag{19}$$

$$G_{offline} = 1 - \exp\left\{-p^{\vartheta}\left[T_{offline}\left(e^{\alpha I + \mu L}\right)^{-\vartheta}\right]\right\} \tag{20}$$

(19) and (20) imply the market share

$$G(p) = 1 - \left(1 - G_{online}(p)\right)\left(1 - G_{offline}(p)\right) \tag{21}$$

$$\lambda_{online} = \int_0^{\infty}\left[1 - G_{offline}(p)\right] dG_{online}(p) = \frac{T_{online}\left(w_{online}\right)^{-\vartheta}}{T_{online}\left(w_{online}\right)^{-\vartheta} + T_{offline}\left(e^{\alpha I + \mu L}\right)^{-\vartheta}} \tag{22}$$

Assume all the products are homogeneous. The online and offline sectors market are clear. Here I use $w = w_{online}$ for concise.

$$\underbrace{\phi w S}_{\text{Wage}} = \underbrace{[\phi S w + (1-\phi)S]\lambda_{online}}_{\text{Consumption}} \tag{23}$$

where ϕ is proportion that individuals work in online sector. The market is incomplete, which means that people work in each sector cannot move into another freely. Only healthy people can work during pandemics and receive wage w. And all these money will be consumed on each time point. The LHS is the total wage of online sector and RHS is total consumption of agents. For offline sector I have

$$\left[\phi S w + (1-\phi)S\right]\left(1 - \lambda_{online}\right) = (1-\phi)S \tag{24}$$

It is a self-consistent market clean condition. From equation (23) the wage is $w = \frac{1-\phi}{\phi}\frac{\lambda}{1-\lambda}$.

3.2 Planning Problem

The total value can be attributed to three parts: (1) ρ is the ratio of production value of online and offline sectors; (2) $(1-\rho)$ is the ratio of value of other sectors accept fixed wage $\bar{w}$; and (3) τ is the preference of consumption, agents in each sector will choose different proportion of goods produced by online or offline sectors;

$$\max_{L} \int_0^\infty e^{-(r+v)t} \left\{ \begin{array}{c} \underbrace{\rho\left[\phi S(t)w + (1-\phi)S(t)\right]}_{\text{Value of Online and Offline Sectors}} \\ + \underbrace{\bar{w}(1-\rho)(1-L(t))[\tau(S(t)+I(t))+1-\tau]}_{\text{Value of Other Sector}} \\ - \underbrace{\chi_d \times I\varphi(I)}_{\text{Loss}} \end{array} \right\} dt \tag{25}$$

Table 1: Parameters Calibration

Parameter	Value	Definition
β	0.2	Daily increase of active cases if unchecked
γ	0.055	Daily rate of infected recovery (includes those that die)
φ	0.01γ	IFR: fatality per active case (per day)
κ	0.05γ	Implies a 3 percent fatality rate with 40 percent infected
r	0.05	Annual interest rate 5 percent
ν	0.667	Prob rate vaccine + cure (exp. duration 1.5 years)
θ	0.5	Effectiveness of lockdown
χ_d	see Section 4	Value of Statistical Life
α	0.5	value of potential price w.r.t. $I(t)$
μ	0.8	value of potential price w.r.t. $L(t)$
$\tilde{\alpha}$	0.99	effectiveness of lockdown
T_i	Offline 2 and Online is 1.5	Type II distribution parameters for sectors

4 Numerical Results

I show the main quantitative results of the planning problem described in the previous section. Throughout my primary focus is on the controlled and uncontrolled scenario with comparing different risk aversion parameters α, value of life χ_d to explore the optimal lockdown policies and changes in output, wages, and government deficits on this basis. I first analyze the one sector situation. And then move to the comparisons under different industrial structures to capture the optimal lockdown policy and their corresponding outcomes. In terms of methodology, I use value function iteration method to solve the HJB equations. In order to use this method, I have to get the difference form of the HJB equations. The process can be seen in the Appendix A. Calibration is shown in Table 1.

4.1 Baseline Model: Risk Aversion and Life Value

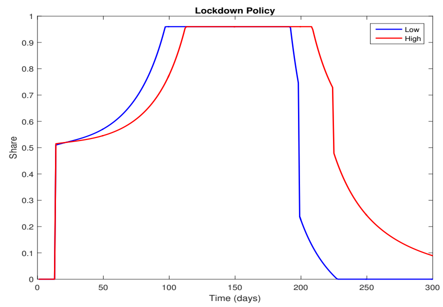
Risk aversion shapes the decision making of individuals. When representative agents take a hedging attitude towards unknown risks, it will directly affect individual decision making. Presented in Figure 1, the optimal lockdown policy monotonously declines after a short period of high lockdown rate. As the degree of risk aversion increases, the lockdown policy becomes stricter. Tighter lockdown policies have led to a decline in output, which ultimately fell by 2.5%. Increased risk aversion also leads to an obvious whole decline in wages. In both risk aversion scenarios, the wages appear to fall first and then rise. The fiscal deficit curves vary in the same trend, and stricter lockdown policy leads to a whole increase in the fiscal deficits. For the yield of the economy, the shock of the pandemic is totally negative for the economy. With the proportion $I(t)$ increases the lost of the yield is also enhanced, from 2% to 2.5%.

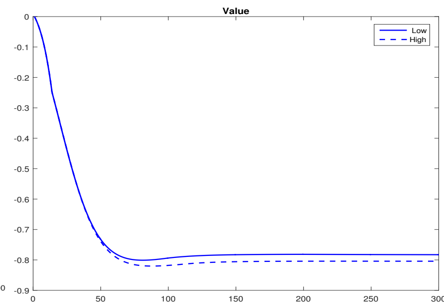
In my baseline model, the wage is same as price index ($w_t = p_t$), so I only need to focus on the variation of wage of working population. I can get the conclusion from the numerical solution that the severity of pandemic is negative correlated with the price index (wage), which means the increasing of $I(t)$ means the minimum of price decreases. In my results, it decreases from 0.981 to 0.977. And then, it will increase and converge to a fixed value. The reason for this can be explained by the fiscal policy dynamics. At the first stage, there are still many people who work on positions and accept the wage w_t. Pandemic shock decreases the wage at first. And then, fiscal policy exerts its ability to meet the requirement and demanding. Along with people who are infected and given unemployment insurance, the demanding increases again.

Fiscal deficit comes from the unemployment policy b. According the setting of the model, it should be concert with the trend of infected people. Deficit increases to 2.25% with the proportion $I(t)$ enhances. And then, the same as wage and yield, decrease to a fixed value.

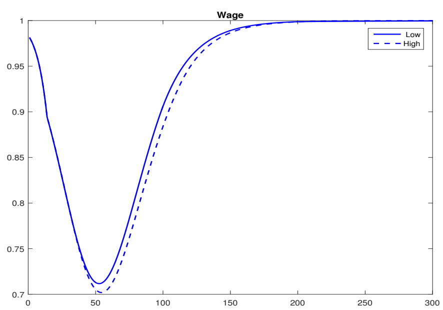
I set $b = 0.05$ and $b = 0.15$ as two comparison. With benefit decreases the yield of economy will also decrease, but fiscal deficit decreases also, as I expected. And the minimum price also increase, which means that a more rigid lockdown policy will be implemented.

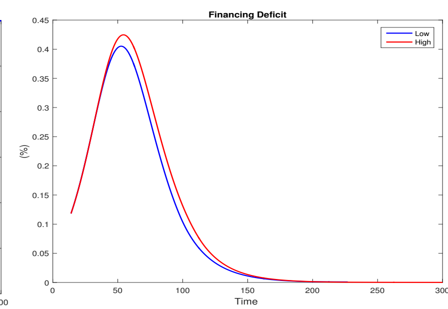
The statistical value of life is self-evident, and as the value of life goes up, I tend to stick to a conservative lockdown policy. Presented in Figure 2, both of the optimal lockdown curves under the two scenarios show the monotonous decline and cross at date 100. As the value of life increases, the lockdown policy becomes stricter, resulting in an overall decline in output. The output curve with $\chi_d = 1.5$ first falls and then rises. The wage curve with $\chi_d = 1.5$ shows an obvious whole decline comparing to the curve with $\chi_d = 0.6$. The fiscal deficit curves first increases and then decreases, and stricter lockdown policy leads to a whole increase in the fiscal deficits.





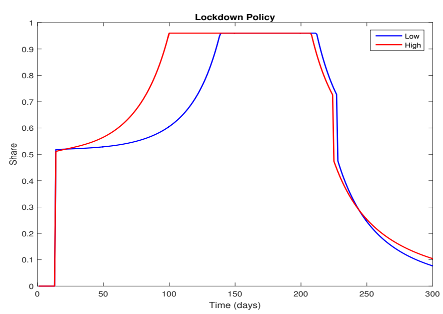
(a) Lockdown (b) Output

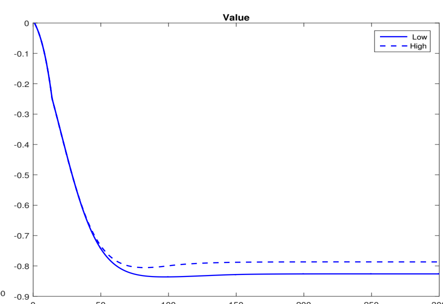




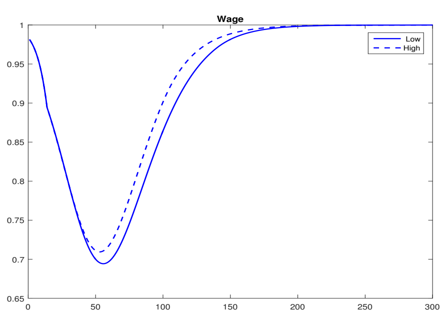
(c) Wage (d) Deficit

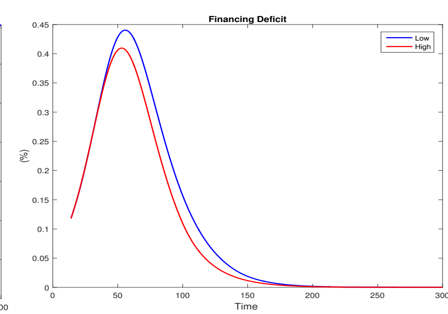
Figure 1: Risk aversion, $\alpha = 0.01$, $\alpha = 0.02$





(a) Lockdown (b) Output





(c) Wage (d) Deficit

Figure 2: Value of Life, $\chi_d = 1.5$ or $\chi_d = 0.6$

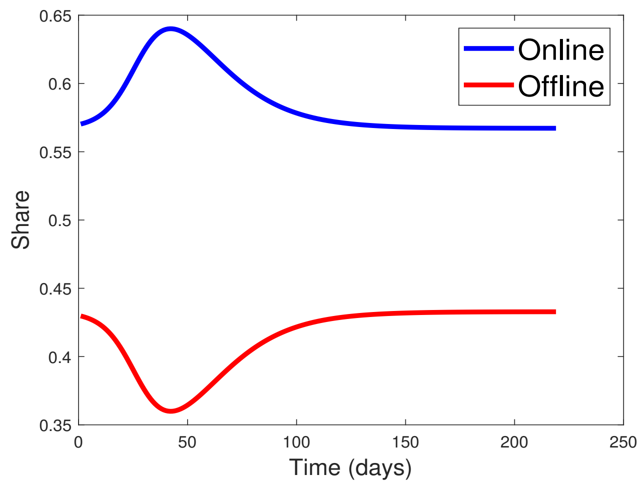


Figure 3: Online and Offline Structure

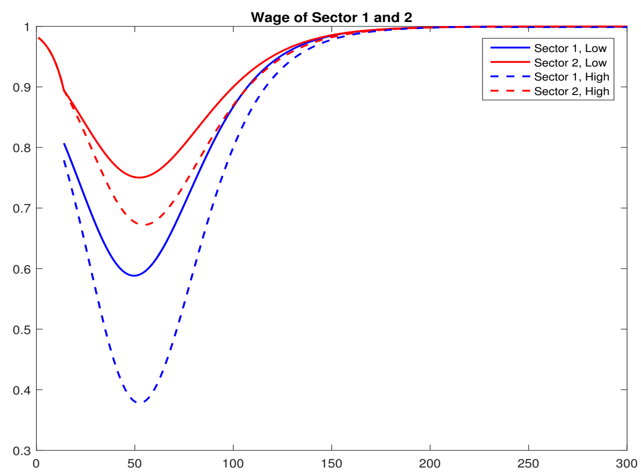


Figure 4: Online and Offline wage inequality

4.2 Online and Offline structure and Inequality

In the figure 3, I show modified industrial structure model to explore how the market share of these two parts will change during the pandemic in the context of lockdown policy. When the lockdown policy is implemented, the market share of offline part will go up and offline sector will be crowed out. After lockdown period, these two market share return to initial state. It expressed the core idea I want to transfer. In Figure 4, I set the same *vsl* value as baseline line value. Wage premium is enlarged with the increasing of *vsl*. With the increasing of *vsl*, optimal lockdown policy is tightened and leads the gap increases as I mentioned before.

5 Conclusion

According to my analysis, I sort out the relationship between lockdown and susceptible, infected population, and lockdown under a framework of general equilibrium. Main insight I provide for this arena is the substantial nexus between industrial structure and the critical index I mentioned above. Different industry organizations can give rise to distinct outcomes for the whole economy. This model dictates the crowding effect of online relative to offline part: when the industrial structure varies largely, the degree of inequality will be dampened significantly because of the increased wage gap.

However, I still do not answer whether the debt will impact welfare in the long term. Since I are mainly considering short-term decisions, and one possible strategy is to integrate long-term and short-term goals to measure this issue. Under the condition of an infinite time limit, the decision is time-homogeneous; In other words, starting at any time, my decision strategy is independent of time and related to other state variables. This may ignore the impact of variable debt accumulated over time. It is possible to approach this problem with a combination of short and long terms. It is not self-evident whether the short-term benefit increase is better or the future benefit loss from debt is more significant.

Besides, there are still some other uncharted fields I want to explore under this framework. The first nature is CRRA utility function, a typical utility function argued in many ex-works. It can reveal risk-aversion identity for inter-temporary decision making. Another problem is CES productive function, which can integrate various intermediate producers into one part flexibly. Considering the heterogeneous-agent nature about age, initial assets, in a word, these natures can describe the economy more realistically since these initial conditions affect the decision making under the pandemic scenario.

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A Appendix for Numerical Method

As Alvarez et al. [2021], I use value function iteration algorithm to solve this problem.
The utility of agents is

$$E[u_s(c_t^s, n_t^s)] = (1-\alpha L)Lw + \alpha Lb \tag{26}$$

because I can calculate the equilibrium wage, the utility can be written as:

$$u_t^s = (1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb \tag{27}$$

Solve this social planner's problem:

$$\max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S(t)\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I(t)b - \chi_d D(t) \right\} dt \tag{28}$$

The HJB equation of this problem is:

$$\begin{aligned}(r+v)V(S,I) = \min_{L\in[0,1]} \Bigg\{ & S(t)\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I(t)b + \chi_d \phi(I(t))I(t) + \\ & - \partial_S V(S,I)[-\beta S(t)I(t)(1-\theta L(t))^2] \\ & + \partial_I V(S,I)[\beta S(t)I(t)(1-\theta L(t))^2 - \gamma I(t)] \Bigg\}\end{aligned} \tag{29}$$

To calculate the partial difference $\partial_S V(S,I)$ and $\partial_I V(S,I)$, I choose to $V_S^-(i,j)$ and $V_I^+(i,j)$

$$V_S^-(i,j) = \frac{V\left(S_i, I_j\right) - V\left(S_{i-1}, I_j\right)}{S_i - S_{i-1}} \tag{30}$$

$$V_I^+(i,j) = \frac{V\left(S_i, I_{j+1}\right) - V\left(S_i, I_j\right)}{I_{j+1} - I_j} \tag{31}$$

$$\begin{aligned}(r+\nu)V\left(S_i, I_j\right) = \min_{L\in[0,1]} & \left\{ S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha Lb\right] + I_j b + \chi_d \phi(I_j) I_j \right. \\ & \left. + \left[\beta S_i I_j (1-\theta L)^2\right]\left[V_I^+(i,j) - V_S^-(i,j)\right] - \gamma I_j V_I^-(i,j) \right\}\end{aligned} \tag{32}$$

I assume that $S_i - S_{i-1} = I_{j+1} - I_j = \Delta$.

$$V_I^+(i,j) - V_S^-(i,j) = \frac{1}{\Delta}\left[V\left(S_i, I_{j+1}\right) - V\left(S_i, I_j\right) - V\left(S_i, I_j\right) + V\left(S_{i-1}, I_j\right)\right] \tag{33}$$

With respect the situation that I set the interval of different of direction on the discrete space.

$$
\begin{aligned}
V\left(S_i, I_j\right) = \min_{L\in[0,1]} & \left\{ S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha L b\right]dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1-(r+\nu)dt]\left\{1 - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}\right\} V\left(S_i, I_j\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j+1}\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} V\left(S_{i-1}, I_j\right) \\
& \left. + [1-(r+\nu)dt]\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j-1}\right) \right\}
\end{aligned} \tag{34}
$$

On the edge of the space:

$$
\begin{aligned}
V\left(S_i, I_j\right) = \min_{L\in[0,1]} & \left\{ S_i\left[(1-\alpha L)\frac{S(1-L)-Ib}{S} + \alpha L b\right]dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1-(r+\nu)dt]\left\{1 - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}\right\} V\left(S_i, I_j\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} V\left(S_{i-1}, I_{j+k}\right) \\
& \left. + [1-(r+\nu)dt]\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j-1}\right) \right\}
\end{aligned} \tag{35}
$$

Before value function iteration, I have to determine the initial value of discrete space:

$$
V(0, I) = \frac{a^2 b^2}{8\alpha\gamma + 4\alpha(r+v)} I^2 + \frac{4\alpha\chi_d\phi + 2ab}{4\alpha\gamma + 4\alpha(r+v)} I + \frac{1}{4\alpha(r+v)} \tag{36}
$$

$$
V(S, 0) = \frac{\left(\frac{(1+\alpha b S)^2}{4\alpha}\right)}{r+v}; \tag{37}
$$

And then I use FOC to find optimal policy $L(t)$

$$
L = \frac{(1 + \alpha b I + a b S) - 2\theta[\beta S I]\left[V_I^+ - V_S^-\right]}{2\alpha - 2\theta^2[\beta S I]\left[V_I^+ - V_S^-\right]} \tag{38}
$$

B Other results

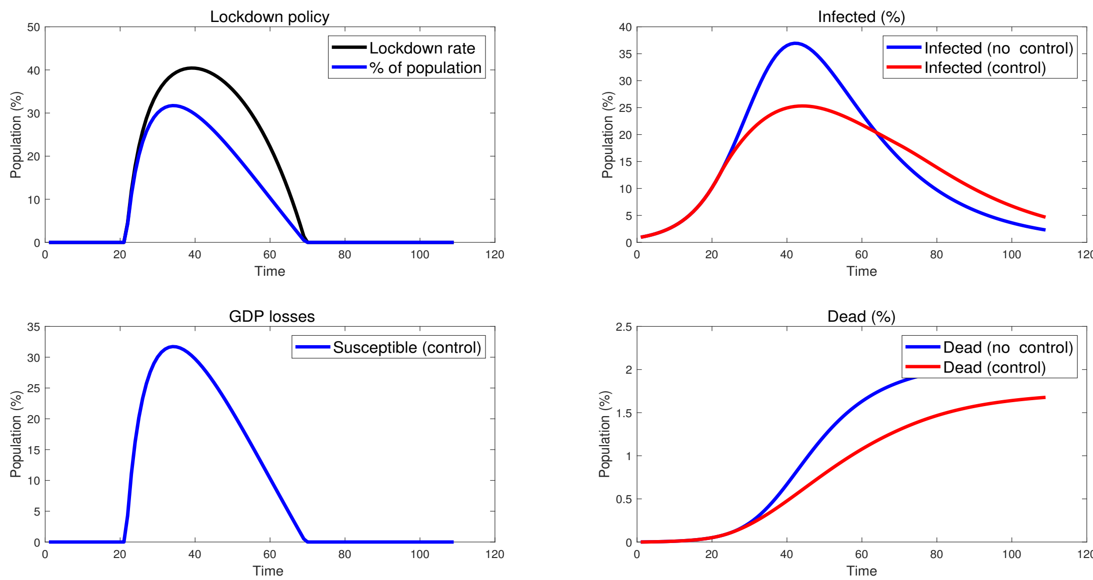

Figure 5: Lockdown Effect under controlled and uncontrolled scenarios. This figure compares the dynamics of population and GDP data in the cases of controlled and uncontrolled scenarios